Quasi volume law of entanglement entropy in nonequilibrium steady states

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Recently, the entanglement entropy S in ground states, i.e. equilibrium states at zero temperature, has been attracting much attention. In particular, its size dependence has been studied intensively. For the ground states of translationally-invariant (extensive) Hamiltonians in one dimension, it was revealed that $S = O(1)$ (i.e., area law) if the excitation spectrum has a finite gap, whereas $S = O(\ln L)$ (logarithmic law) for gapless systems of length L [1]. However, the case of nonequilibrium steady states (NESSs) is almost unknown, even at zero temperature.

To reveal the size dependence of S in NESSs, we study a mesoscopic conductor driven by electron reservoirs, using a well-established model that reproduces many experimental results. The model is a non-interacting electron system on a one-dimensional chain composed of a conductor of finite length and two reservoirs of semi-infinite length. There is a random potential of mean strength W in the conductor region, which causes multiple scatterings of electrons and the wavefunctions of electrons have complicated shapes.

We assume zero temperature for the reservoirs. Then, the total quantum state is simply given by a single Slate determinant of left- and right-coming single-particle states, which are occupied up to the two Fermi seas of left and right reservoirs, respectively. Since the total quantum state is pure, its entanglement entropy is the von Neumann entropy. The difference $\Delta\mu$ of the chemical potentials of the two Fermi seas induces a finite current in the conductor, and a NESS is realized. Using this state, we calculate S in the conductor, and examined its L dependence.

In equilibrium states ($\Delta\mu = 0$) without random potential ($W = 0$), we obtain $S = O(\ln L)$, in consistency with the previous result [2,3]. The same bahavior is obtained for equilibrium states ($\Delta\mu = 0$) with random potential ($W > 0$).

However, the behavior changes dramatically in NESSs ($\Delta\mu > 0$) with random potential ($W > 0$). In this case we find an anomalous increase of S; $S = L\eta(L) + O(\ln L)$, which we call the quasi-volume law. Here, $\eta(L)$ is a positive function gradually decreasing with increasing L. Moreover, we find that the quasi-volume law appears only when both $\Delta\mu$ and W are non-vanishing. i.e., only when the driving force and multiple scatterings coexist.

We clarify the physical origin of the quasi-volume law.

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